# Characteristic Frequency Calculation of Double-Bass-reflex Loudspeaker Enclosure

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## Preface

This document stresses procedure to calculate characteristic frequencies of double-bass-reflex enclosure. Procedure we see here is not newly developed model; however, I lost reference link made by some one else. Theoretical procedure presented here is consistent with technical documents published throughout http://mcap.webcrow.jp/.

## 1. Traditional Formulae Review

Nagaoka indicated the formula without reference[1]. According to current Wikipedia in Japanese, the formula was developed by Nagaoka himself.

Fig.1 shows typical configuration of double-bass-reflex enclosure. Table 1 gives definitions of symbols. These symbols are used throughout this document, but traditional formulae uses different unit system and it is noted in this paragraph.

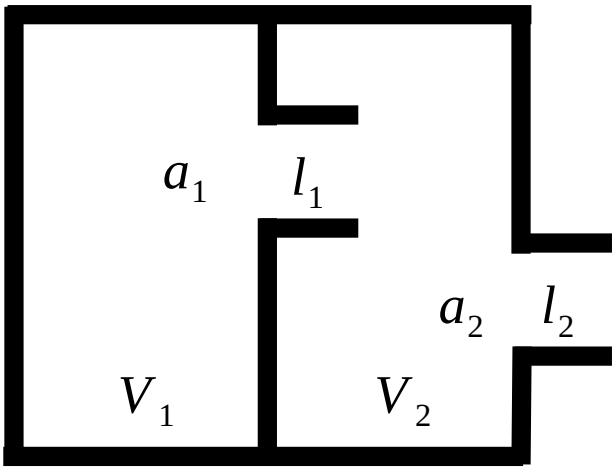


Fig.1 Resonant Model

Table 1 List of Symbols

| symbol | description | unit | remark |
|---|---|---|---|
| $V_1$ | #1 chamber volume | $m^3$ | |
| $V_2$ | #2 chamber volume | $m^3$ | |
| $a_1$ | #2 duct cross-sectional area | $m^2$ | |
| $a_2$ | #2 duct cross-sectional area | $m^2$ | |
| $l_1$ | #1 duct length | m | |
| $l_2$ | #2 duct length | m | |

Nagaoka indicated formulae in the reference. We see the formulae in modified form in order to clarify physical interpretation. Following formulae (1) and (2) are modified forms. They assume (a) and (b) below. Note that unit system is not indicated in the reference book, so that we discuss unit system in this paragraph.

$$f_{d1}=160\sqrt{\frac{a_1}{V_1 l_1}}\sqrt{\frac{V_1+V_2}{V_2}} \tag{1}$$

$$f_{d2}=160\sqrt{\frac{a_2}{l_2(V_1+V_2)}} \tag{2}$$

Assumptions of above formulae:

(a) Two chambers works separately for higher characteristic frequency.
(b) Two chambers are combined and assumed as single to calculate lower characteristic frequency.

Vibration model of double-bass-reflex is equivalent to following spring-mass model (Fig.2).

1 Nagaoka, “20 Selected Newest Original Speaker Designs by Nagaoka Tetsuo”, Ongakunotomosha, 1987, ISBN4-276-24030-1 C0073 P1550E

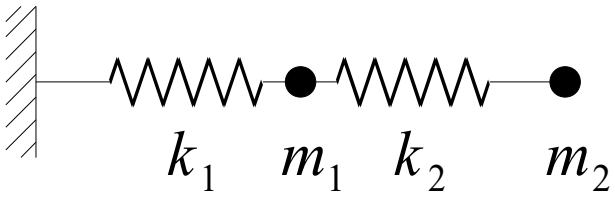


Fig.2 Equivalent Spring-Mass model of Double-Bass-reflex Enclosure

This is not too difficult model to solve theoretically. We discuss theoretical model later.
Interpretation of assumptions are simple. We see the following equivalent spring-mass models below:

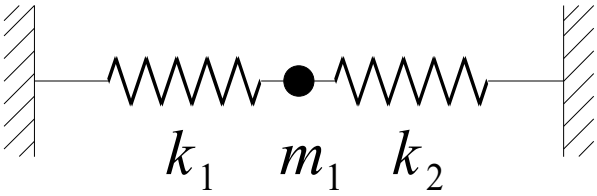


Fig.2a Equivalent Model of Assumption (a)

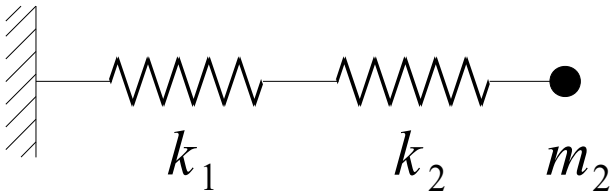


Fig.2b Equivalent Model of Assumption (b)

We notice that assumptions (a) and (b) are different systems and inconsistent.

In any way we see how the equation (1) and (2) are organized with assumptions (a) and (b).

Equation (1) and (2) are not easily interpreted, therefore we compare with theoretical equations based on thermodynamics.

$$f_{d1}=\frac{1}{2\pi}\sqrt{\frac{k_1+k_{2(1)}}{m_1}} \qquad (1)'$$

$$f_{d2}=\frac{1}{2\pi}\sqrt{\frac{1}{m_2}\left(\frac{1}{k_1}+\frac{1}{k_{2(2)}}\right)^{-1}} \qquad (2)'$$

We keep in mind that $k_2$ is not unique value for chamber-duct vibration system: $k_2$ is function of cross-sectional are of applied duct.

For $m_1$, we have following relationship[2]:

$$k_{2(1)}=\frac{\gamma a_1^2 P}{V_2} \qquad (3).$$

In the same way, for $m_2$, we have following relationship:

$$k_{2(2)}=\frac{\gamma a_2^2 P}{V_2} \qquad (4).$$

2 Refer to document ID MCAP001E of http://mcap.webcrow.jp/documents.html.

Masses $m_1$ and $m_2$ are expressed as below:

$$m_1 = \rho a_1 l_1 \tag{5}$$

$$m_2 = \rho a_2 l_2 \tag{6}$$

Substituting equations (3) - (6) for equation (1)', we get:

$$f_{d1} = \frac{1}{2\pi}\sqrt{\frac{\gamma P}{\rho}}\sqrt{\frac{a_1}{V_1 l_1}}\sqrt{\frac{V_1 + V_2}{V_2}} \tag{7}$$

In the same manner, substituting equations (3) - (6) for equation (2)', we get:

$$f_{d2} = \frac{1}{2\pi}\sqrt{\frac{\gamma P}{\rho}}\sqrt{\frac{a_2}{l_2(V_1 + V_2)}} \tag{8}$$

where,

| | |
|---|---|
| $P$ | : atmospheric pressure (=101,300[Pa]) |
| $\gamma$ | : ratio of specific heat of air $c_p / c_v$ (=1.4 for adiabatic condition) |
| $\rho$ | : density of air (=1.2[kg/m$^3$] |
| other variables | : refer to Table 1. |

Using SI unit system indicated by Table 1. Coefficient becomes:

$\frac{1}{2\pi}\sqrt{\frac{\gamma P}{\rho}}$ =54.7 for adiabatic condition, and $\frac{1}{2\pi}\sqrt{\frac{\gamma P}{\rho}}$ =46.2 for isothermal condition.

Because the coefficient does not match the traditional formulas', we assume that traditional formulae use following units:

| | |
|---|---|
| $V$ | : [l] |
| $l$ | : [cm] |

Then coefficient becomes $\frac{1}{2\pi}\sqrt{\frac{10\gamma P}{\rho}}$ instead of $\frac{1}{2\pi}\sqrt{\frac{\gamma P}{\rho}}$ .

Now coefficient $\frac{1}{2\pi}\sqrt{\frac{10\gamma P}{\rho}}$ becomes 173 for adiabatic condition, and 146 for isothermal condition.

Coefficient indicated by the traditional formulae is 160 and it is between 173 and 146. It seems that experimental modification was applied to theoretical value.

Now verification of traditional formulae is complete. Then we discuss theoretical formulae in the next paragraph.

## 2. Theoretical Formulae

Equations of motion of the bass-reflex enclosure under free vibration condition is

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} = \begin{bmatrix} k_1 + k_2\dfrac{a_1^2}{a_2^2} & -k_2\dfrac{a_1}{a_2} \\ -k_2\dfrac{a_1}{a_2} & k_2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{9}$$

where,

$x$ : Duct displacement ".." above stands for 2nd-order derivative by time.

Each symbol is defined already in the previous paragraph.

Substituting above symbols for equation (1), then we get :

$$\begin{bmatrix} \rho a_1 l_1 & 0 \\ 0 & \rho a_2 l_2 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} = \gamma P \begin{bmatrix} \dfrac{a_1^2}{V_1} + \dfrac{a_1^2}{V_2} & -\dfrac{a_1 a_2}{V_2} \\ -\dfrac{a_1 a_2}{V_2} & \dfrac{a_2^2}{V_2} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Dividing both sides by ρ, we get:

$$\begin{bmatrix} a_1 l_1 & 0 \\ 0 & a_2 l_2 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} = \frac{\gamma P}{\rho} \begin{bmatrix} \dfrac{a_1^2}{V_1} + \dfrac{a_1^2}{V_2} & -\dfrac{a_1 a_2}{V_2} \\ -\dfrac{a_1 a_2}{V_2} & \dfrac{a_2^2}{V_2} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{10}$$

Characteristic equation is:

$$\begin{vmatrix} a_1^2\left(\dfrac{1}{V_1} + \dfrac{1}{V_2}\right) - \dfrac{\rho a_1 l_1 \lambda}{\gamma P} & -\dfrac{a_1 a_2}{V_2} \\ -\dfrac{a_1 a_2}{V_2} & \dfrac{a_2^2}{V_2} - \dfrac{\rho a_2 l_2 \lambda}{\gamma P} \end{vmatrix} = 0 \tag{11}$$

We define β for simplicity.

$$\beta = \frac{\rho}{\gamma P}$$

Rewriting equation (11) with β, we get:

$$\begin{vmatrix} \dfrac{a_1^2(V_1 + V_2)}{V_1 V_2} - \beta a_1 l_1 \lambda & -\dfrac{a_1 a_2}{V_2} \\ -\dfrac{a_1 a_2}{V_2} & \dfrac{a_2^2}{V_2} - \beta a_2 l_2 \lambda \end{vmatrix} = 0 \tag{12}$$

Equation (12) is rewritten as

$$\beta^2 a_1 a_2 l_1 l_2 \lambda^2 - \beta \frac{a_1^2 a_2 l_2 (V_1 + V_2) + a_1 a_2^2 l_1 V_1}{V_1 V_2}\lambda + \frac{a_1^2 a_2^2 (V_1 + V_2)}{V_1 V_2^2} - \frac{a_1^2 a_2^2}{V_2^2} = 0 \tag{13}$$

Dividing both sides by $a_1 a_2$ we get

$$\beta^2 l_1 l_2 \lambda^2 - \beta \frac{a_1 l_2 (V_1 + V_2) + a_2 l_1 V_1}{V_1 V_2} \lambda + \frac{a_1 a_2}{V_1 V_2} = 0 \tag{14}$$

Equation (14) can be exactly solved with root formula below:

$$a\lambda^2 + b\lambda + c = 0 \quad \text{where,} \quad a \neq 0$$

Then we have following root formula:

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4\mathrm{ac}}}{2\mathrm{a}} \tag{15}$$

We can calculate constants $a$ , $b$ , and $c$ as follows:

$$a = \beta^2 l_1 l_2$$
$$b = -\beta \frac{a_1 l_2 (V_1 + V_2) + a_2 l_1 V_1}{V_1 V_2}$$
$$c = \frac{a_1 a_2}{V_1 V_2}$$

Characteristic frequency is expressed using eigen value as:

$$f_D = \frac{1}{2\pi} \sqrt{\lambda} \tag{16}$$

Because formulae with two equations are too complicated, we have calculation sheet with above procedure in appendix.

## Appendix

# Characteristic Frequency Calculation Sheet(REV00:2018-11-16)

How to use this calculation sheet (double-bass-reflex only)

Put your design value to the cells.(V1, V2, A1, A2, L1, L2)

Any cell in the color left shall not be overw ritten.

Duct length correction model is not incorporated to this sheet. Users shall modify duct length before using this sheet.

Chamber volumes

| | [litre] | [m3] | |
|---|---|---|---|
| V1 | 6.09 | 0.00609 | Primary chamber |
| V2 | 20.4 | 0.0204 | Secondary chamber |
| V1+V2 | 26.49 | 0.02649 | Total |

| Calculation Results | | |
|---|---|---|
| Characteristic frequencies | Corrected Values* | Existing Formula |
| fd1 | **110** | 104 |
| fd2 | **54** | 57 |

* Multiplied by 0.925 to adiabatic model results.

Cross-sectional area of ducts

| | [cm2] | [m2] | |
|---|---|---|---|
| A1 | 25 | 0.0025 | Primary duct |
| A2 | 30.25 | 0.003025 | Secondary duct |

Length of ducts

| | [mm] | [m] | [cm] | |
|---|---|---|---|---|
| L1 | 126 | 0.126 | 12.6 | Primary duct |
| L2 | 90 | 0.09 | 9 | Secondary duct |

| | | | |
|---|---|---|---|
| γ | ratio of specific heat | 1.4 | [-] |
| ρ | density of air | 1.2 | [kg/m3] |
| P | air pressure | 101300 | [Pa] |
| β | defined here | 8.4614E-06 | [m-4 s2] |

stiffness matrix of chamber system

| | column1 | column2 |
|---|---|---|
| raw1 | 157.496 | -43.812 |
| raw2 | -43.812 | 53.012 |

coefficients of root formula

| | | |
|---|---|---|
| a | | 8.1190E-13 |
| b | | -5.6403E-07 |
| c | | 6.0872E-02 |
| sqrt(D) | sqrt(b^2-4ac) | 3.4705E-07 |

Eigenvalues of equations of motions

| | | |
|---|---|---|
| λ1 | eigen value 1 | 5.6108E+05 |
| λ2 | eigen value 2 | 1.3363E+05 |

| | characteristic frequencies | theoretical values w ith adiabatic conditions | Corrected values* | |
|---|---|---|---|---|
| fd1 | Frequency 1 | 119.3 | 110 | 0.9248554913 |
| fd2 | Frequency 2 | 58.2 | 54 | |