Simplified Method to Estimate Characteristic Frequencies of MCAP-CR

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Multiple Chamber Aligned in Parallel Cavity Resonator (MCAP-CR) Speaker Systems have multiple characteristic frequency and therefore can extend lower frequency range. On the other hand, it requires solving Eigen value problems. It was somewhat difficult except for engineers or scientists.

It is discussed to approximate characteristic frequencies without solving Eigen value problem.

[1] Simplification of Equation of Motions of Masses in Ducts

We may solve up to 2 degree of freedom equations without using calculation problems as discussed in MCAP001E. In case both mass matrix and stiffness matrices are diagonal, all the mass moves independently, so that we can break the series of equation to independent equations; however, for MCAP-CR, some of non-diagonal elements are not zero, so that calculation program was required.

On the other hand, I became to think that there should be some other simplified calculation may be possible to estimate characteristic frequencies. Here, we discuss simplified method to estimate characteristic frequency.

(1) Equation of 1 degree of freedom problem

Stiffness of air, mass of air involved in the duct, characteristic frequencies are calculated as follows:

Stiffness of air in the chamber chamber $$k = \frac{\gamma \cdot a^2 P}{V} \text{ [N/m]} \tag{1}$$

Mass of air $$m = \rho \cdot a \cdot l \text{ [kg]} \tag{2}$$

where,

- ρ : Density of air at atmospheric pressure [kg/m^3]
- a: Cross sectional area of the duct [m^2]
- l : Equivalent length of the duct[m]
- γ : Ratio of specific heat Cp/Cv(=1.4)[1]
- P: Atmospheric pressure (=101,300[Pa])

Resonant frequency $$f_D = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{\gamma \cdot a \cdot P}{\rho \cdot l \cdot V}} \quad \text{[Hz]} \tag{3}$$

Basic idea to simplify the calculation is to separate variables, while variable are not

[1] We use equithermal condition instead of adiabatic condition, based on experimental results, therefore we let γ =1.0.

separated. This is not scientifically true, but it will be proven that it is practical enough below.

(2) Hypothesis of simplified calculation

Fig.1 shows typical structure of MCAP-CR where number of sub-chambers is 4.

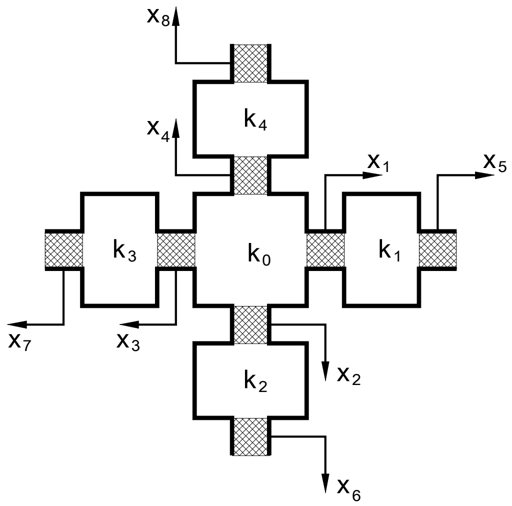

Fig.1 Free Vibration Model of MCAP-CR (n=4)

Stiffness of air in each chamber is a function of cross-sectional area of each duct. We use stiffness value for reference area for simplicity. Reference area is chosen as membrane area. Actual stiffness for a duct is expressed using ration of duct area divided by reference area. Reference area is defined as a_0 and ratio is defined as r_j. Let us define volume in each chamber be: $V_1, V_2, ..., V_n$, and cross sectional area of each duct be: $a_1, a_2, ..., a_n, a_{n+1}, ..., a_{2n}$. Then stiffness of each chamber for each duct is expressed in equation (4).

Suffix j is defined as each duct which connects main chamber and sub chamber. Suffix that is greater than n is defined as open-air duct.

$$k_j^* = \frac{r_j^2 a_0^2 P}{V_j} \tag{4}$$

where, $$r_j = \frac{a_j}{a_0} \tag{5}$$

Table 1 gives stiffness of air in each chamber.[2]。

[2] Refer to MCAP001E for definition of symbols.

Table 1 Volume and Stiffness of Each Chamber (j=1, ...,n)

	Volume[m^3]	Stiffness for a_0 [N/m]	Stiffness for a_j [N/m]
Main Chamber	V_0	$k_0 = \dfrac{a_0^2 P}{V_0}$	$k_{0j}^* = \dfrac{r_j^2 a_0^2 P}{V_0}$
Sub Chamber(j)	V_j	$k_j = \dfrac{a_0^2 P}{V_j}$	$k_j^* = \dfrac{r_j^2 a_0^2 P}{V_j}$
Sub Chamber(n+j)	V_{j+n}	$k_{n+j} = \dfrac{a_0^2 P}{V_j}$	$k_{n+j}^* = \dfrac{r_{n+j}^2 a_0^2 P}{V_j}$

(3) Calculation of Characteristic Frequency Based on Simplified Assumptions

We assume the followings:

(a) Each mass of each duct moves independently[3].

(b) Some part of next chamber works as a part of referenced chamber[4].

(c) Effect of chambers beyond next chamber is ignored.

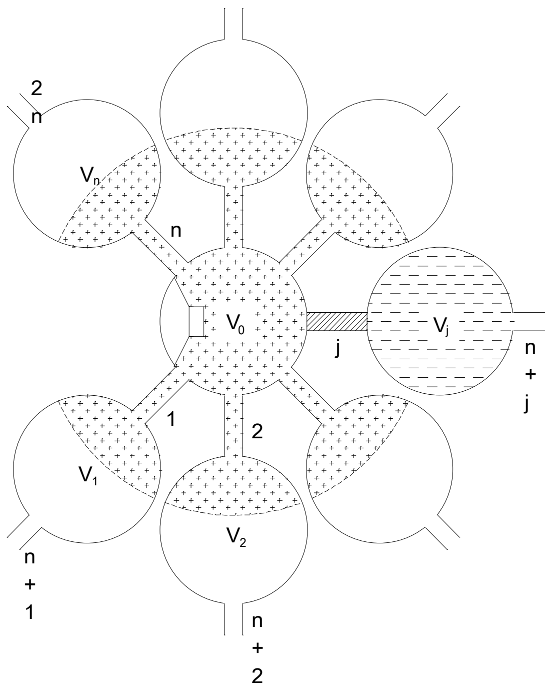

Fig.2a

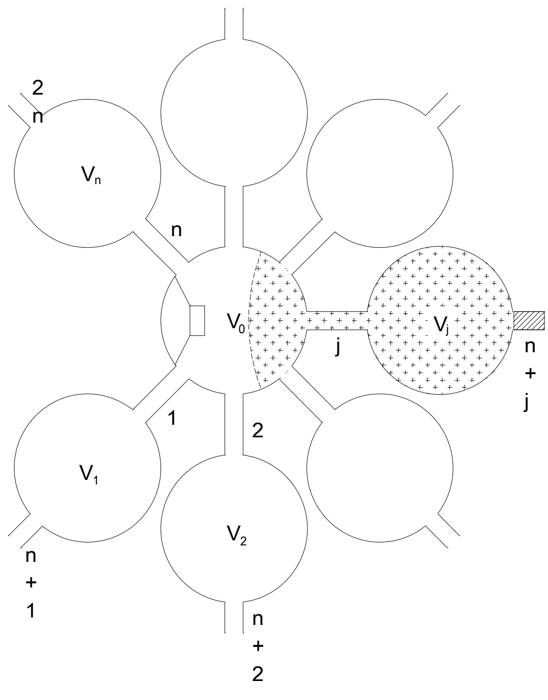

Fig.2b

Fig.2a、2b expressed assumptions (b) - (c).

Fig.2a references duct "j". If this duct moves to right hand side, chamber Vj pushes back and chamber Vo pulls back. Some part of chambers, V1, V2,...,Vn, except Vj works as part of V0. Fig.2breferences duct "n+j". Some part of V0 acts as part of Vj. It seems that Vj becomes larger.

[3] This is not true, because there are mutual interactions by function of each chamber; however, vibration equations may be superposed as approximation.

[4] This is different from Equation model of MCAP-CR. Mass of air is not rigid, so that this assumption may be better than exact model of MCAP-CR.

Table 2 gives stiffness of air in each chamber, mass of air involved in each duct, and characteristic frequencies based on assumption (1) - (c).

Table 2 Calculation Formulae of Characteristic Equations Based on Simplified Assumption

	Equivalent Volume[m³]	Equivalent Stiffness [N/m]	Mass[kg]	Characteristic Frequencies[Hz]
Inter -chamber duct j=1, ...,n	$\hat{V}_0 = V_0 + \sum_{i=0}^{n} \alpha_i V_i - V_j$ $\hat{V}_j = \dfrac{1}{\dfrac{1}{\hat{V}_0} + \dfrac{1}{V_j}}$	$\hat{k}_j^* = \dfrac{r_j^2 a_0^2 \gamma \cdot P}{\hat{V}_j}$	$m_j = \rho \cdot r_j a_0 l_j$	$\hat{f}_j = \dfrac{1}{2\pi}\sqrt{\dfrac{\hat{k}_j^*}{m_j}}$
Open -air duct j=1, ...,n	$\hat{V}_j = V_j + \beta_j V_0$	$\hat{k}_{j+n}^* = \dfrac{r_{j+n}^2 a_0^2 \gamma \cdot P}{\hat{V}_j}$	$m_{j+n} = \rho \cdot r_{j+n} a_0 l_{j+n}$	$\hat{f}_{j+n} = \dfrac{1}{2\pi}\sqrt{\dfrac{\hat{k}_{j+n}^*}{m_{j+n}}}$

^ stands for modified value based on the assumptions, α_j , β_j are parameters that estimate effect of next chamber. It is also assumed that $0 \le \alpha_j \le 1$ and $0 \le \beta_j \le 1$. Value of the parameter 1 means that next chamber is completely involved in the referenced chamber, and 0 means that next chamber does not work at all.

[2] Comparison Between Simplified Calculation Results and Experimental Results

Each result is compared to assure if these assumptions are practical enough. It was not easy to determine all characteristic frequencies from impedance trend curve, so I determined characteristic frequencies from response curve, although local response peaks may not represent characteristic values.

Experiment was done using model TR130b with model L5 full-range driver by Omnes Audio. Please note that driver is different from Application Report's. Full-range drive W5-1611SA by Tangband was finally chosen for model TR130b.

Table 3 gives specification of model TR130b enclosure and Fig.3 shows drawing of model TR130b enclosure.

Refer to Appendix-2 of document MCAP001E how characteristic frequencies were calculated originally.

Table-3 Specification of Model TR130b Enclosure

Item	Specification	Characteristic Frequency Determined by Calculation Programme[5]
Main Chamber Volume	9.0[L]	
#1 Sub-Chamber Volume	9.7[L]	
#2 Sub-Chamber Volume	9.6[L]	
#3 Sub-Chamber Volume	10.5[L]	
#1 Duct (H x W x L)	□40mm×40mm	94Hz
#1 Duct (H x W x L)	□40mm×80mm	85Hz
#2 Duct (H x W x L)	□40mm×100mm	67Hz
#3 Duct (H x W x L)	□33mm×100mm	49Hz
#4 Duct (H x W x L)	□33mm×180mm	37Hz
#5 Duct (H x W x L)	□33mm×260mm	30Hz

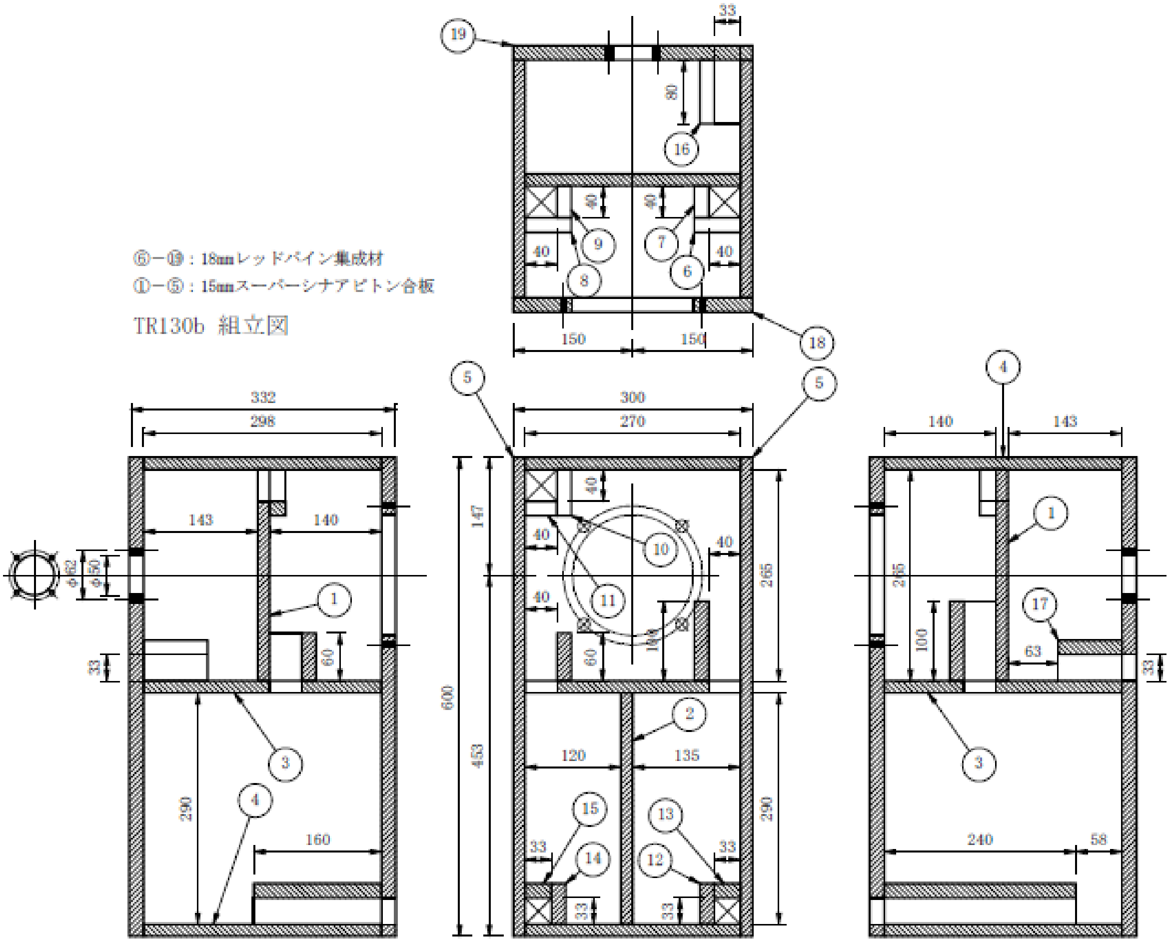

Fig.3 Mechanical Drawing of Model TR130b Enclosure

Tuning parameters αj and βj were chosen to minimize differences from previous calculation results. Table 4 gives calculation results of simplified method and comparison with measured and designed values.

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[5] Characteristic frequencies are not ducts' own property, but simultaneous characteristic frequencies of MCAP-CR system.

Table-4 Comparison Between Measured Results and Calculation Results

Duct	α_j	β_j	m_j [kg]	$\hat{k}_j$ [N/m]	$\widehat{f}_j$ [Hz]	f_j^P [Hz]	f_j^E [Hz]	Note
#1	1	-	0.0000768	37.73	111.6	94	96.4	β is not used
#2	1	-	0.0001536	35.62	76.6	85	84.2	β is not used
#3	1	-	0.0001920	33.86	66.8	67	70.0	β is not used
#4	-	0	0.0001307	13.35	50.8	49	52.4	α is not used
#5	-	0	0.0002352	12.38	35.6	37	41.9	α is not used
#6	-	0	0.0003398	11.44	29.2	30	31.0	α is not used

$\widehat{f}_j$: Characteristic frequencies using simplified approach

f_j^P : Characteristic frequencies using calculation programme (MCAP001E)

f_j^E : Estimated characteristic frequencies from observed response

Fig. 4 shows frequency response curve of this experiment. Yellow circles are supposed peaks of characteristic frequencies, and light green circles were other local peaks.

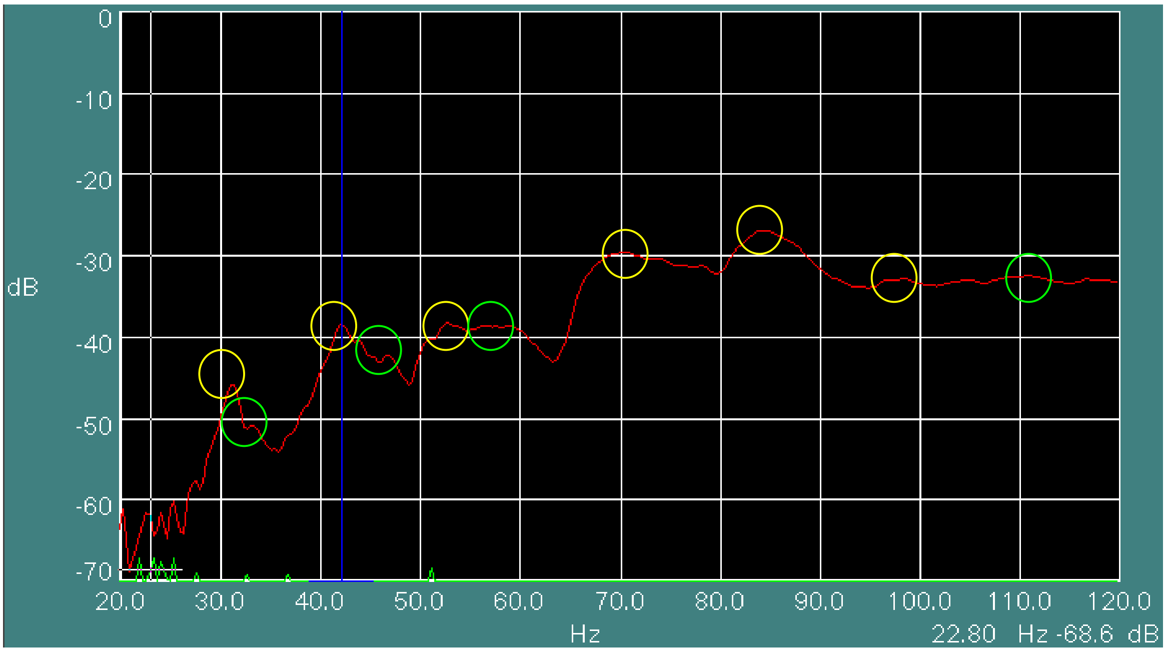

Fig.4 Frequency Response Using Log-sweep Signal

[3] Analyses

Although experimental results do not guarantee true characteristic frequencies, errors by calculation programme (Appendix-B of MCAP001E) are in the range of [-3Hz, +5Hz]. This result seems to be practical enough. On the other hand, errors using simplified approach are larger (+19.8Hz/-7.6Hz), especially in the higher frequencies. Errors in lower three characteristic reside in the range of +1.6Hz/+6.3Hz. These errors seem to be practically acceptable in actual designing.

These results are interpreted as follows:

Pressure wave in the back of membrane goes through main chamber and drives inter-chamber ducts. Phase of characteristic frequencies of inter-chamber ducts or higher frequency turns to be reverse phase and it works as bass-reflex, phase of lower frequency passes through inter-chamber ducts at same phase but drives open-air ducts then phase becomes reverse and works as bass reflex enclosure. In this case, inter-chamber ducts function as membranes and open-air ducts work as ducts of cavity resonators. This explains that β was always 0. In addition to this resonant mode, there seems a little effect of next chamber. It means main β many not be zero for this mode. It may be the reason that MCAP-CR can generate lower frequency than minimum characteristic frequency.

The most difficult estimation is the higher characteristic frequencies. The closest estimation was got where α =1. This estimation may not be said as accurate enough; however, this frequency range may be considerably radiated from front face of membrane, so we could say this is practically accurate.

[4] Conclusion

These simplified estimation formulae are practically accurate enough. We may use these formulae given in Table 2 to estimate characteristic frequencies of MCAP-Cars.

End of Report