Structure and Momentum Equations of CBS-CR

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1. Overview

CBS-CR was propsed as one of typical multiple-degree of freedom cavity resonator system. CBS-CR is suitable to layout air chambers in a plain.
Assuming rectangular main air-chamber, then four surfaces face subchambers. In the same way, subchambers face another subchambers. This is similar to carbon molecule which has four bonding hands, therefore, I named this structure CBS-CR (carbon bond structured cavity resonator).
Although application examples of CBS-CR are few, I found that CBS-CR can extend lowest frequency end compared with standard MCAP-CR of same size. And also I did not feel negative effects of CBS-CR structure. Hence I decided to develop CBS-CR simulator, modifying the latest MCAP-CR steady-state simulator application. It will help ones who try CBS-CR applications.

2. Structures of CBS-CR

Typical CBS-CR are are classified to three types as Fig. 1a - 1c. shows. Type CS can be extended to bigger degrees of freedom, but it will not be practical.

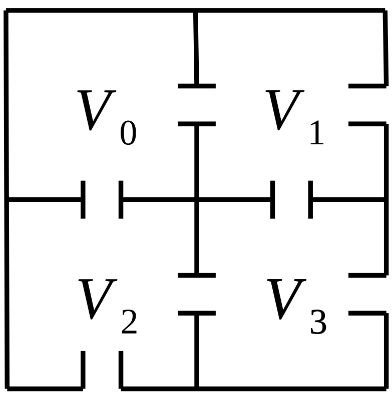

Fig. 1a. Type CQ

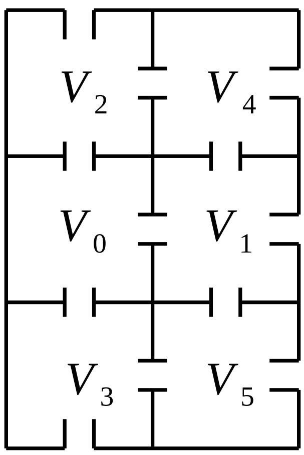

Fig. 1b. Type CH

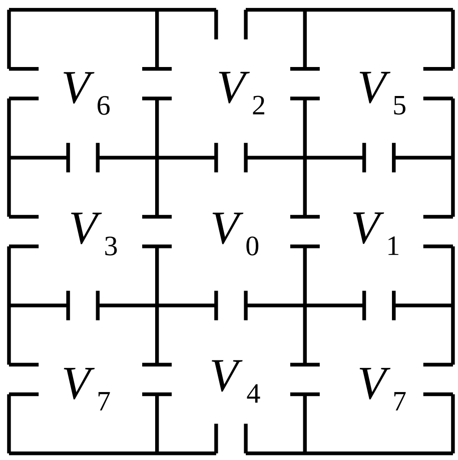

Fig. 1c. Type CS

V_0 in Fig.1a-1c indicates main chamber where speaker driver is installed. Other chambers are sub-chambers, which are connected to main or other sub-chambers with ducts. These chambers and ducts configure spring-mass vibration system. Table 1 gives summary of each type.

Table 1 Structural Summary of CBS-CR

	Type CQ	Type CH	Type CS
Number of Main chambers	1	1	1
Number of Chambers	4	6	9
Number of Internal Ducts	4	7	12
Number of External Ducts	3	5	8
Maximum Number of Degree of Freedom	9	13	21

CBS-CR has greater degrees of freedom, because it is more complex than standard MCAP-CR.

3. Equations of Motion of CBS-CR

Equations of motions of CBS-CR may be derived in the same manner as MCAP-CR. Refer to related white papers in the site for more details.

3.1 General Form - Equations of Motion

$$\boldsymbol{M}\ddot{\boldsymbol{x}}+\boldsymbol{C}\dot{\boldsymbol{x}}+\boldsymbol{K}\boldsymbol{x}=\boldsymbol{f}$$

where,

- $\boldsymbol{M}$ mass matrix
- $\boldsymbol{C}$ friction coefficient matrix
- $\boldsymbol{K}$ stiffness matrix
- $\boldsymbol{x}$ displace ment vector (dot above x indicates derivative with respect to time).
- $\boldsymbol{f}$ external force (driving force) vector

quantity of each mass

$$m_j=\rho a_j l_j$$

where,

- ρ density of air[kg/m^3]
- a_j cross sectional area of each duct[m^3]
- l_j length of each duct[m]

standard stiffness of each chamber (for reference piston area)

$$k_j=\frac{\gamma a_0^2 p_0}{V_j}$$

where,

- γ specific heat ratio of air(= $\frac{C_p}{C_v}$)
- a_0 effective piston area of the driver (reference area value)[m^2]
- p_0 atmospheric pressure(=101.3[kPa])

area ratio of each duct and area ratio matrix

$$r_j=\frac{a_j}{a_0}$$

$$\boldsymbol{R}=(\delta_{ij} r_j)$$

actual stiffness matrix (for actual ducts)

$$\boldsymbol{K}=\boldsymbol{R}\hat{\boldsymbol{K}}\boldsymbol{R}$$

where,

- $\hat{\boldsymbol{K}}$ standard stiffness matrix

3.2 Standard Stiffness Matrix

Standard stiffness matrices were delivered for Types CQ, CH and CS. Refer to Appendix-1 for actual matrix.

Continues to Appendix-1

Appendix -1 Standard Stiffness Matrices of CBS-CR Types CQ, CH and CS

Here are standard stiffness matrices of CBS-CR Types C, B and A. Blank cells mean zero.

Type CQ Standard Stiffness Matrix

	0	1	2	3	4	5	6	7
0	k_u+k_0	k_0	k_0					
1	k_0	k_0+k_1	k_0	$-k_1$		$-k_1$		
2	k_0	k_0	k_0+k_2		$-k_2$		$-k_2$	
3		$-k_1$		k_1+k_3	k_3	k_1		$-k_3$
4			$-k_2$	k_3	k_2+k_3		k_2	$-k_3$
5		$-k_1$		k_1		k_1		
6			$-k_2$		k_2		k_2	
7				$-k_3$	$-k_3$			k_3

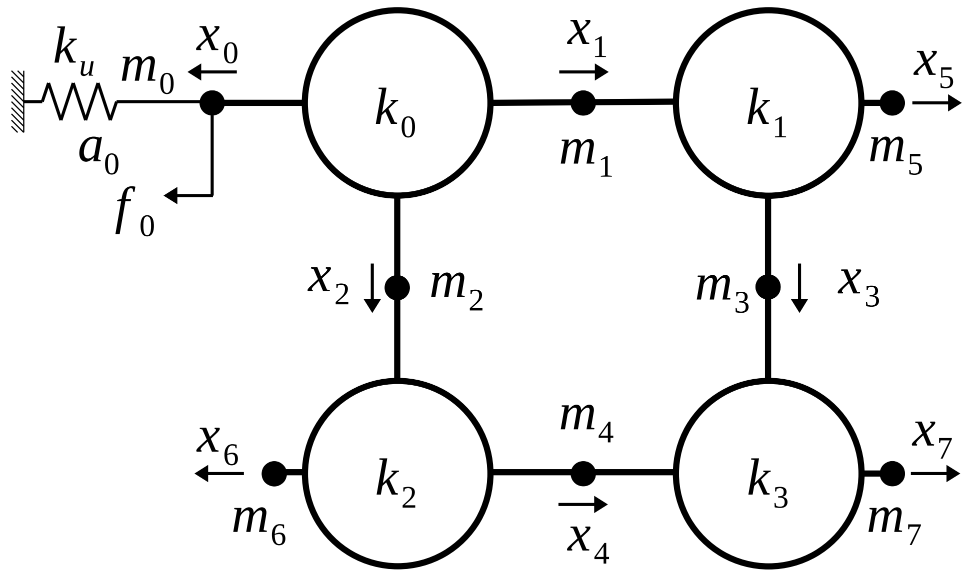

Fig. A-1 Numbering Rule of Type CQ

Type CH Standard Stiffness Matrix

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	k_u+k_0	k_0	k_0	k_0									
1	k_0	k_0+k_1	k_0	k_0	$-k_1$	$-k_1$			$-k_1$				
2	k_0	k_0	k_0+k_2	k_0			$-k_2$			$-k_2$			
3	k_0	k_0	k_0	k_0+k_3				$-k_3$			$-k_3$		
4		$-k_1$			k_1+k_4	k_1	k_4		k_1			$-k_4$	
5		$-k_1$			k_1	k_1+k_5		k_5	k_1				$-k_5$
6			$-k_2$		k_4		k_2+k_4			k_2		$-k_4$	
7				$-k_3$		k_5		k_3+k_5			k_3		$-k_5$
8		$-k_1$			k_1	k_1			k_1				
9			$-k_2$				k_2			k_2			
10				$-k_3$				k_3			k_3		
11					$-k_4$		$-k_4$					k_4	
12						$-k_5$		$-k_5$					k_5

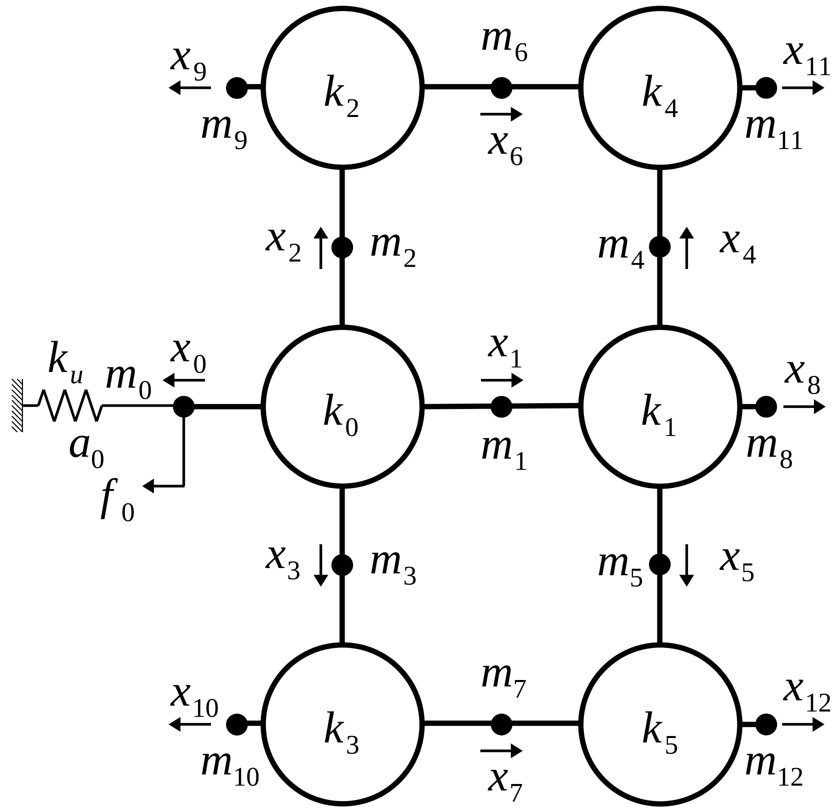

Fig.A-2 Numbering Rule of Type CH

Type CS Standard Stiffness Matrix

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0	k_u+k_0	k_0	k_0	k_0	k_0																
1	k_0	k_0+k_1	k_0	k_0	k_0	$-k_1$	$-k_1$							$-k_1$							
2	k_0	k_0	k_0+k_2	k_0	k_0			$-k_2$	$-k_2$						$-k_2$						
3	k_0	k_0	k_0	k_0+k_3	k_0					$-k_3$	$-k_3$					$-k_3$					
4	k_0	k_0	k_0	k_0	k_0+k_4							$-k_4$	$-k_4$				$-k_4$				
5		$-k_1$				k_1+k_5	k_1	k_5						k_1				$-k_5$			
6		$-k_1$				k_1	k_1+k_8						k_8	k_1							$-k_8$
7			$-k_2$			k_5		k_2+k_5	k_2						k_2			$-k_5$			
8			$-k_2$					k_2	k_2+k_6	k_6					k_2				$-k_6$		
9				$-k_3$					k_6	k_3+k_6	k_3					k_3			$-k_6$		
10				$-k_3$						k_3	k_3+k_7	k_7				k_3				$-k_7$	
11					$-k_4$						k_7	k_4+k_7	k_4				k_4			$-k_7$	
12					$-k_4$		k_8					k_4	k_4+k_8				k_4				$-k_8$
13		$-k_1$				k_1	k_1							k_1							
14			$-k_2$					k_2	k_2						k_2						
15				$-k_3$						k_3	k_3					k_3					
16					$-k_4$							k_4	k_4				k_4				
17						$-k_5$		$-k_5$										k_5			
18									$-k_6$	$-k_6$									k_6		
19											$-k_7$	$-k_7$								k_7	
20							$-k_8$						$-k_8$								k_8

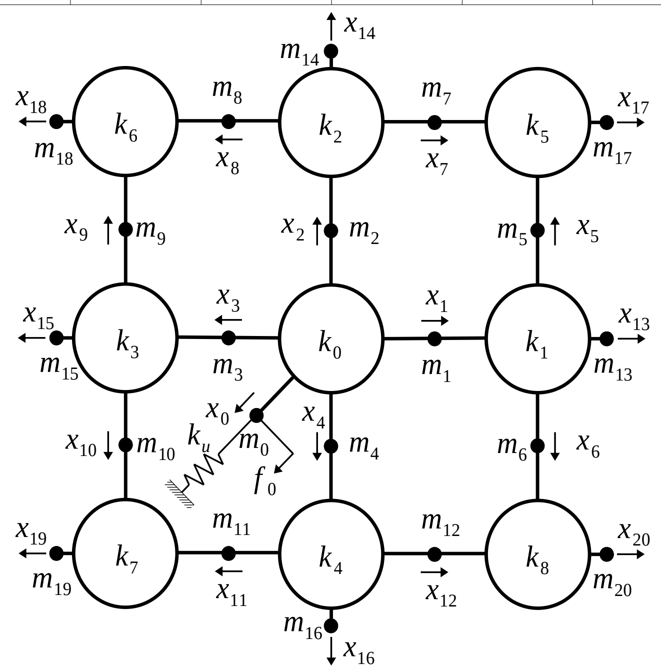

Fig.A-3 Numbering Rule of Type CS