Numerical Simulation of Standard MCAP-CR Loudspeaker System

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1. Preface

We have discussed characteristic frequencies[ii] and procedure[iii] to solve equations of motion of MCAP-CR. We recall that form of equations in matrix format is uniform so that determining stiffness matrix will make it possible to solve any type of multiple degree of freedom cavity resonator applications. We focus on standard MCAP-CR applications in this report. We discuss the procedure how to solve equation of motion numerically.

2. Discretised Equation of Motion of Standard MCAP-CR

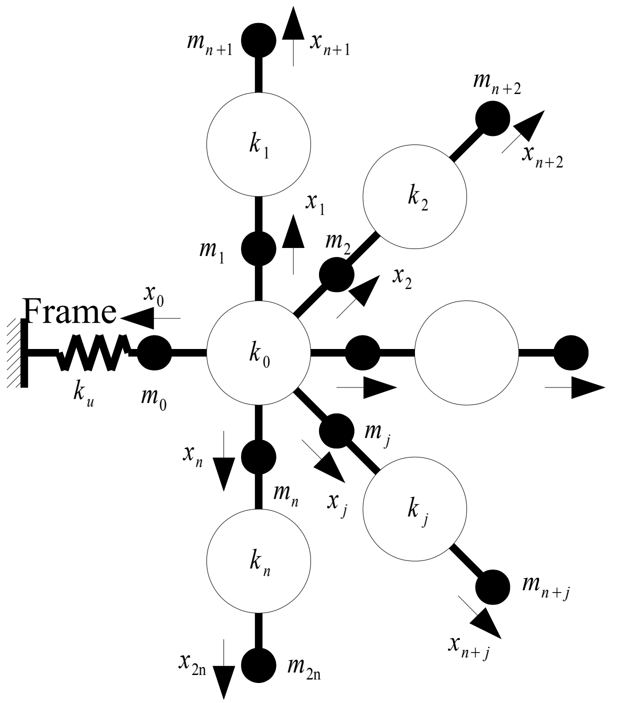

Fig. 1 Definition of Symbols and Directions

Equation of motion of any cavity resonator application in matrix form is expressed as

$$\boldsymbol{M}\frac{d^2\boldsymbol{x}}{dt^2}+\boldsymbol{C}\frac{d\boldsymbol{x}}{dt}+\boldsymbol{K}\boldsymbol{x}=\boldsymbol{f} \quad (1)$$

where,

- **M** : mass matrix [kg]
- **C** : damping matrix [kg/s]
- **K** : stiffness matrix [N/m]
- **f** : external force vector [N]
- **x** : displacement of mass vector[m].

Central difference equation form of equation (1) is

$$\boldsymbol{M}\frac{\boldsymbol{x}^{j+1}-2\boldsymbol{x}^{j}+\boldsymbol{x}^{j-1}}{\delta^2}+\boldsymbol{C}\frac{\boldsymbol{x}^{j+1}-\boldsymbol{x}^{j-1}}{2\delta}+\boldsymbol{K}\boldsymbol{x}^{j}=\boldsymbol{f}^{j} \quad (2)$$

where,

- j, j+1, j-1 : discrete time
- δ : interval of discrete time.

We ignore damping effect for simplicity then equation (1) is simplified as

$$\boldsymbol{M}\frac{d^2\boldsymbol{x}}{dt^2}+\boldsymbol{K}\boldsymbol{x}=\boldsymbol{f} \quad (3).$$

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i Input file in calculation design sheet in Appendix was integrated to single file.

ii Suzuki, “Equations to Calculate Characteristic Frequencies of Multiple Chamber Aligned in Parallel Cavity Resonator (MCAP-CR)”, 2008

iii Suzuki, “Simulation of Cavity Resonator Systems”, 2008

Recursive form of difference equation (3) is

$$\boldsymbol{x}^{j+1} = (2\boldsymbol{I} - \delta^2 \boldsymbol{M}^{-1}\boldsymbol{K})\boldsymbol{x}^j - \boldsymbol{x}^{j-1} + \delta^2 \boldsymbol{M}^{-1}\boldsymbol{f}^j \quad (4)$$

where,

I : unit matrix.

Next, we assure each matrix values. We assume number of subchambers is two for simplicity.

Mass matrix of standard MCAP-CR with two subchambers is

$$\boldsymbol{M} = \begin{bmatrix} m_0 & 0 & 0 & 0 & 0 \\ 0 & m_1 & 0 & 0 & 0 \\ 0 & 0 & m_2 & 0 & 0 \\ 0 & 0 & 0 & m_3 & 0 \\ 0 & 0 & 0 & 0 & m_4 \end{bmatrix} \quad (5).$$

Stiffness matrix of standard MCAP-CR is expressed as

$$\boldsymbol{K} = \boldsymbol{R}\hat{\boldsymbol{K}}\boldsymbol{R} \quad (6)$$

where,

$$\hat{\boldsymbol{K}} = \begin{bmatrix} k_u + k_0 & k_0 & k_0 & 0 & 0 \\ k_0 & k_0 + k_1 & k_0 & -k_1 & 0 \\ k_0 & k_0 & k_0 + k_2 & 0 & -k_2 \\ 0 & -k_1 & 0 & k_1 & 0 \\ 0 & 0 & -k_2 & 0 & k_2 \end{bmatrix} \text{ and } \boldsymbol{R} = \begin{bmatrix} r_0 & 0 & 0 & 0 & 0 \\ 0 & r_1 & 0 & 0 & 0 \\ 0 & 0 & r_2 & 0 & 0 \\ 0 & 0 & 0 & r_3 & 0 \\ 0 & 0 & 0 & 0 & r_4 \end{bmatrix} \text{ where, } r_j = \frac{a_j}{a_0}.$$

External force may be in any form. Here we assume sinusoidal wave form for simplicity. Please note that Fourier proved that any wave form can be expressed as superposition of sinusoidal waves.

External force is assumed as

$$f_0(t) = b\sin(2\pi F t) \text{ and } f_1(t) = f_2(t) = \dots f_n(t) = 0 \quad (7)$$

where,

b : amplitude of force[N]

F : frequency of force wave[Hz].

Now we get recursive formula of standard MCAP-CR with two subchambers is as follows:

$$\begin{bmatrix} x_0^{j+1} \\ x_1^{j+1} \\ x_2^{j+1} \\ x_3^{j+1} \\ x_4^{j+1} \end{bmatrix} = 2\begin{bmatrix} x_0^{j} \\ x_1^{j} \\ x_2^{j} \\ x_3^{j} \\ x_4^{j} \end{bmatrix} - \delta^2 \boldsymbol{M}^{-1}\boldsymbol{R}\begin{bmatrix} k_u+k_0 & k_0 & k_0 & 0 & 0 \\ k_0 & k_0+k_1 & k_0 & -k_1 & 0 \\ k_0 & k_0 & k_0+k_2 & 0 & -k_2 \\ 0 & -k_1 & 0 & k_1 & 0 \\ 0 & 0 & -k_2 & 0 & k_2 \end{bmatrix}\boldsymbol{R}\begin{bmatrix} x_0^{j} \\ x_1^{j} \\ x_2^{j} \\ x_3^{j} \\ x_4^{j} \end{bmatrix} - \begin{bmatrix} x_0^{j-1} \\ x_1^{j-1} \\ x_2^{j-1} \\ x_3^{j-1} \\ x_4^{j-1} \end{bmatrix} + \begin{bmatrix} \frac{\delta^2 b}{m_0}\sin(2\pi F\delta j) \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (8).$$

2. Solving Discretised Equations

We assume that initial position and velocity of each mass is zero, i.e.

$$\boldsymbol{x}^{-1} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ . \\ . \\ 0 \end{bmatrix} \quad \text{and} \quad \boldsymbol{x}^{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ . \\ . \\ 0 \end{bmatrix} \quad (9).$$

Setting design values of the standard MCAP-CR loudspeaker system, we are ready to calculate move of each mass.

3. Example of Analysis

We see result of analysis of application TR130c as an example. TR130c is equipped with Feastrex's NF5Ex; however, specification of Tangband's W5-1611SA was used instead, because I was not sure about specification of former. Table 1 gives used values in this simulation.

Table 1 Condition of Numerical Analysis

Speaker Driver			Chamber			Duct			Other		
Item	Value	Note	Item	Value	Note	Item	Value	Note	Item	Value	Note
m_0 a_0 f_0	5.76[g] 94 $[cm^2]$ 60[Hz]		V_0 V_1 V_2 V_3	15[L] 10[L] 14[L] 16[L]		$a_1 \times l_1$ $a_2 \times l_2$ $a_3 \times l_3$ $a_4 \times l_4$ $a_5 \times l_5$ $a_6 \times l_6$	20.25 x 50 20.25 x 92 20.25 x 110 12.96 x 120 12.96 x 150 12.96 x 240	$cm^2 \times mm$	Thermal condition amplitude of force	isothermal 1[N]	

Displacement of each mass is expressed as line chart for various frequencies.

Fig.2 and 3, respectively, show displacement of each mass at 20Hz and 27Hz.

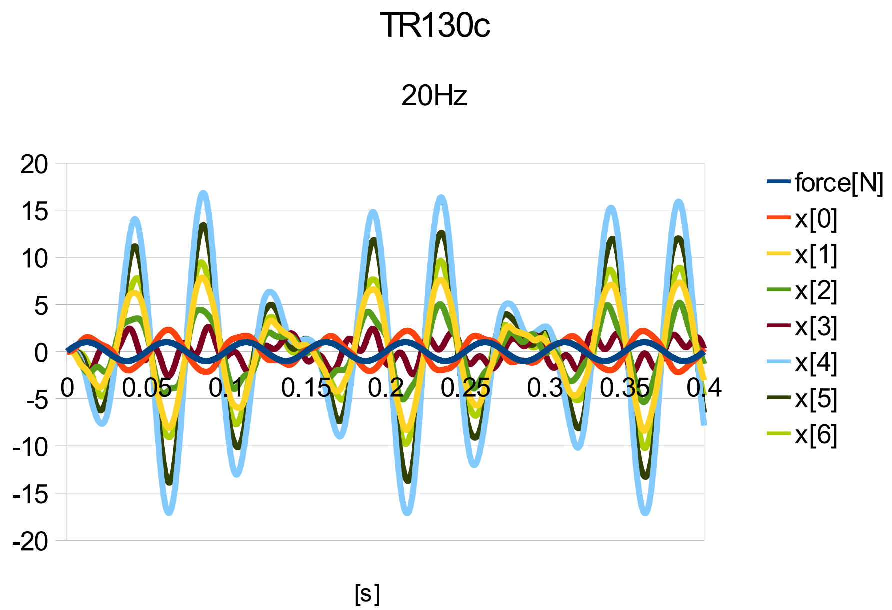

Fig.2

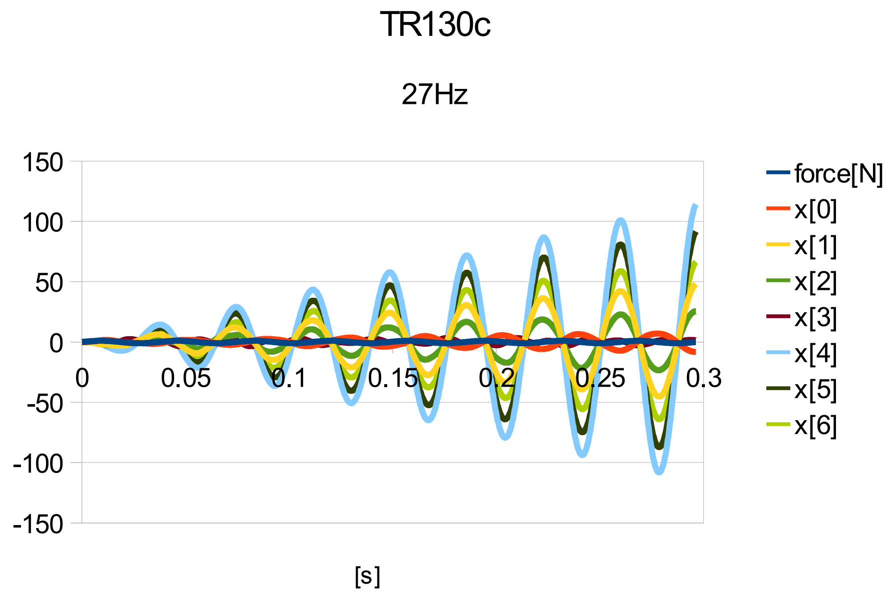

Fig.3

20Hz is beyond specification of this model, so that membrane and most ducts move in the different directions. 27Hz is the lowest end in the specification, so that amplitudes of ducts are much greater than 20Hz's. On the other hands, directions of membrane and ducts are different as well as 20Hz.

There are some other different moves from expected or designed. Duct #6 (see x[6]) is the longest, and volume of installed chamber#6 has the largest volume, so that it was expected that x[6] has the greatest move of all, while x[4] has greater move.

Fig.4 and 5, respectively, show displacement of each mass at 32Hz and 50Hz. Sound pressure level at 32Hz - 100Hz is, as far as I heard, enough.

Fig 4 shows that at least four ducts contributes to increase sound pressure level. 50Hz is more complex than lower frequencies. Internal #3 duct moves different direction from others. Open-air #6 duct, that is on the other side of #3, looks supporting 50Hz.

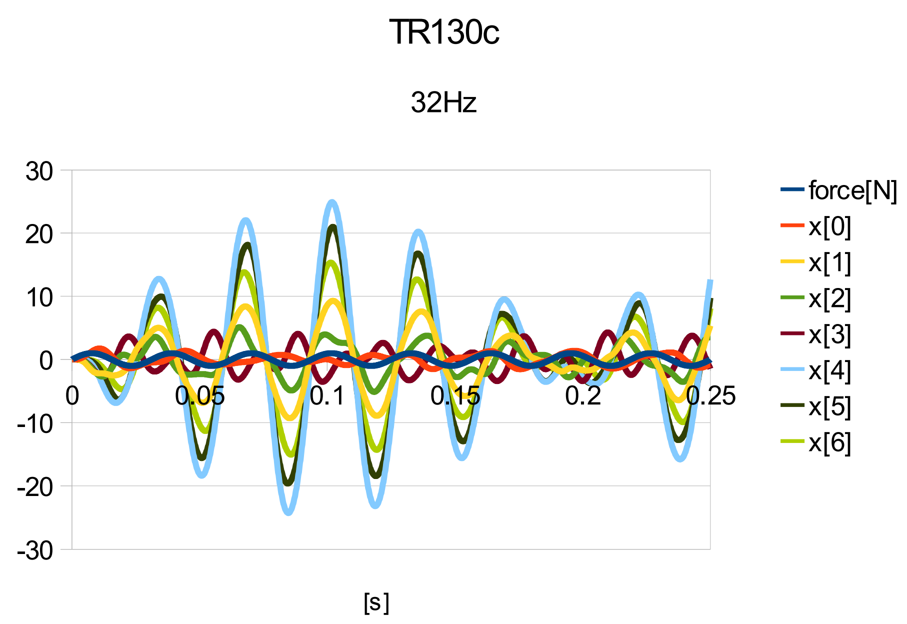

Fig.4

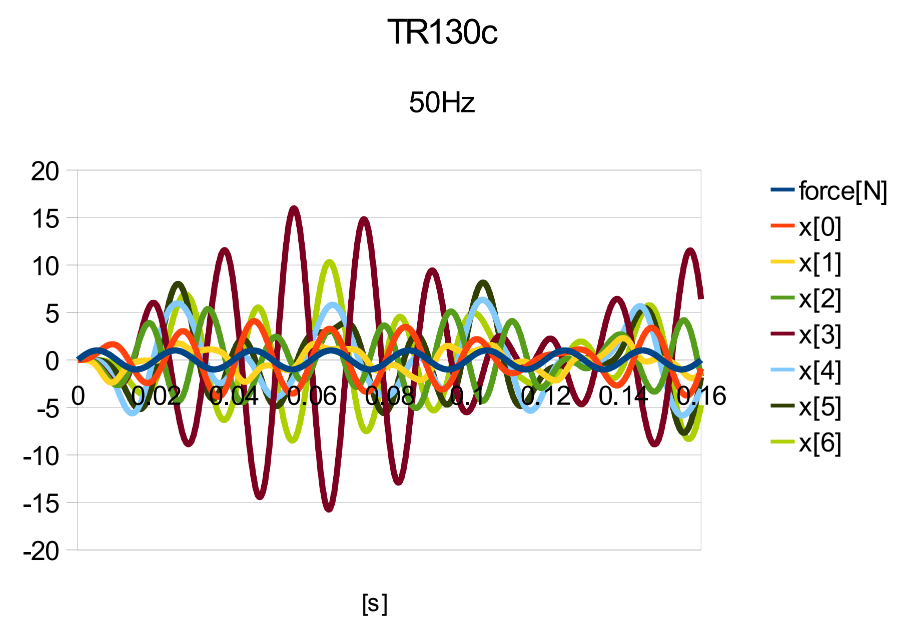

Fig.5

4. Summary

We reviewed procedure of numerical analysis of standard MCAP-CR and some of the result. What we assured was that move of standard MCAP-CR was more complex than expected. It means we need some more research in order to conclude.

I believe it is the first approach to analyze true multiple degree of freedom cavity resonator systems. This will contribute developing high quality loudspeaker systems.

Appendix Calculation Program Design Sheet

<table>
<tr><th>Item</th><th>Specification</th><th>Note</th></tr>
<tr><td>Resolution of calculation</td><td>8bit</td><td>$\delta = \frac{1}{2^8} T = \frac{1}{2^8 F}$ [s]</td></tr>
<tr><td>Calculation Range</td><td>0[s] - n_cycles x T[s]</td><td>$T = \frac{1}{F}$ [s]</td></tr>
<tr><td>External Force</td><td>Frequency F[Hz]
Amplitude b[N]</td><td>$f_0^j = b \sin(2\pi F \delta j)$ $f_i^j = 0$ (i=1,2,...,2n)</td></tr>
<tr><td>Constants</td><td>Atomospheric pressure p0=101300[Pa]
Circular constant pi=3.141592</td><td></td></tr>
<tr><td>Intermediate variables</td><td>Effective are of membrane $a_0[m^2]$
Spring constant of driver $k_u[N/m^2]$</td><td>In case only radius is given like Fostex
$k_u = 4\pi^2 f_0^2 m_0$</td></tr>
<tr><td>Separation character of input files</td><td>TAB(\t)</td><td>Better human interface</td></tr>
<tr><td>Input files</td><td>conditions.txt
Input Format
(\n: after last item, \t: between items)

m0 radius f0
n n_cycles division HCR b
V[0] V[2] … V[n]
A[1] A[2] … A[n] A[n+1] … A[2n]
L[1] L[2] … L[n] L[n+1] … L[2n]</td><td>
<table>
<tr><th>Symbol</th><th>Description</th><th>Unit</th><th>Type</th></tr>
<tr><td>m0</td><td>mass of membrane</td><td>[g]</td><td>double</td></tr>
<tr><td>radius</td><td>effective radius of membrane</td><td>[cm]</td><td>double</td></tr>
<tr><td>f0</td><td>characteristic frequency of driver</td><td>[Hz]</td><td>double</td></tr>
<tr><td>V[i]</td><td>volume of a chamber</td><td>[L]</td><td>double</td></tr>
<tr><td>A[i]</td><td>cross sectional area of a duct</td><td>[cm2]</td><td>double</td></tr>
<tr><td>L[i]</td><td>length of a duct</td><td>[mm]</td><td>double</td></tr>
<tr><td>n</td><td>number of subchambers</td><td>[-]</td><td>int</td></tr>
<tr><td>n_cycles</td><td>number of calculation cycles</td><td>[-]</td><td>int</td></tr>
<tr><td>division</td><td>number of divisions in a cycle</td><td>[-]</td><td>int</td></tr>
<tr><td>HCR</td><td>thermodycamic condition (1: isothermal, 1.4: adiabatic)</td><td>[-]</td><td>int</td></tr>
<tr><td>b</td><td>amplitude of external force</td><td>[N]</td><td>double</td></tr>
</table>
</td></tr>
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