Simulation of Cavity Resonator Systems

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Released in Japanese 2008/5/6
Revised in Japanese 2008/5/24
Released in English 2009/8/9

Cavity Resonator is the most popular Speaker System. It is the most practical in that it is easy to design and cost effective. On the other hand, we do not see many researches of cavity resonator systems. You will see easy way to simulate move of masses of cavity resonators.

[1] Equations of Motion and Their Solution of Single and Extended Cavity Resonators

Any types of cavity resonators including Multi-Degree of Freedom (MDOF) systems like MCAP or MCAS have similar equations of motion. Equations of motion of any kind of cavity resonator system are expressed as the following vector-matrix form:

$$\boldsymbol{M}\ddot{\boldsymbol{x}}+\boldsymbol{C}\dot{\boldsymbol{x}}+\boldsymbol{K}\boldsymbol{x}=\boldsymbol{f}(t) \qquad (1)$$

x is displacement vector of masses, **M** is mass matrix, **C** is damping term matrix, and **K** is stiffness matrix in equation (1)

"●" and "●●" above displacement vector denote respectively derivative and second derivative of time. Right hand side of equation (1) denotes forced vibration vector.

Damping term may be difficult to determine, but other terms are not difficult to determine. Deference formula of equation (1) is:

$$\boldsymbol{M}\frac{\boldsymbol{x}^{j+1}-2\boldsymbol{x}^{j}+\boldsymbol{x}^{j-1}}{\delta^2}+\boldsymbol{C}\frac{\boldsymbol{x}^{j-1}-\boldsymbol{x}^{j-1}}{2\delta}+\boldsymbol{K}\boldsymbol{x}^{j}=\boldsymbol{f}(\omega\cdot\delta\cdot j) \qquad (2)$$

δ denotes discretized time element, and j denotes number of descretized steps. Ignoring damping term, then equation (2) becomes to:

$$\boldsymbol{x}^{j+1}=(2\boldsymbol{E}-\delta^2\boldsymbol{M}^{-1}\boldsymbol{K})\boldsymbol{x}^{j}+\boldsymbol{x}^{j-1}+\delta^2\boldsymbol{M}^{-1}\boldsymbol{f}(\omega\cdot\delta\cdot j) \qquad (3)$$

We used central difference formula as below in equation (3) :

$$\frac{d^2\boldsymbol{x}}{dt^2}\simeq\frac{\boldsymbol{x}^{j+1}-2\boldsymbol{x}^{j}+\boldsymbol{x}^{j-1}}{\delta^2}$$

$$\frac{d\boldsymbol{x}}{dt}\simeq\frac{\boldsymbol{x}^{j+1}-\boldsymbol{x}^{j-1}}{2\delta}$$

We assume initial conditions of equation (3) and calculate one step ahead using recursive formula, then we can calculate movement of each mass.

[2] Forced Vibration Model of Cavity Resonator

Cavity resonator speaker system is expressed in simple spring-mass model like Fig. 1.

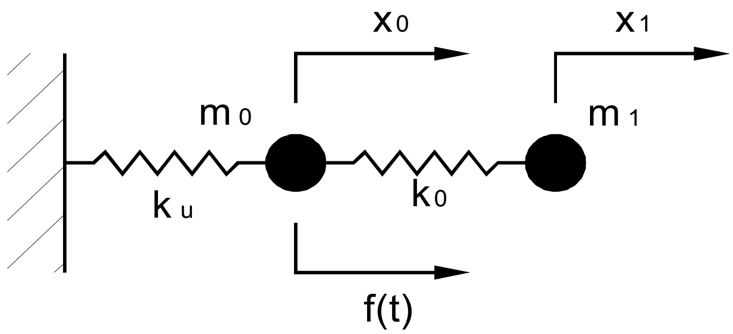


Fig. 1 Equivalent Mass-Spring Model of Cavity Resonator Loudspeaker System

● in the left of Fig.1 is mass of speaker membrane and ● in the right is mass of air involved in duct. k_u is spring constant of speaker driver itself. One end of this spring is rigidly fixed to frame (left hand side of Fig.1). k_0 corresponds to spring constant of air in the chamber; however, k_0 is function of area of moving mass, so k_0 is defined as spring constant corresponding to speaker membrane and k_{01} is defined as spring constant corresponding to duct mass.

Parameters and variables used in Fig.1 are defined as follows:

k_u:	Spring constant of speaker driver [N/m]
k_0:	Spring constant of chamber corresponding to speaker membrane [N/m]
a_0:	Effective membrane area [m^2]
a_1:	Sectional area of duct [m^2]
r_1:	Ratio (a_1/a_0)
m_0:	Effective moving mass of membrane (general definition) [kg]
m_1:	Effective moving mass of air in duct [kg]
f(t):	Driving force by power amplifier [N]
x_0:	Displacement of membrane [m] (Arrow defines positive direction)
x_1:	Displacement of air in the duct [m] (Arrow defines positive direction)

Equation of motion of forced vibration is expressed in (1). We ignore damping term for simplicity, then we get equation (4).

$$\begin{cases} m_0 \dfrac{d^2 x_0}{dt^2} + k_u x_0 + k_0 x_0 - k_0 \dfrac{a_1}{a_0} x_1 = f(t) \\ m_1 \dfrac{d^2 x_1}{dt^2} + k_0 \dfrac{a_1^2}{a_0^2} x_1 - k_0 \dfrac{a_1^2}{a_0^2} \dfrac{a_0}{a_1} x_0 = 0 \end{cases} \qquad (4)$$

We arrange equation (4) and get (4)'.

$$\begin{cases} m_0 \dfrac{d^2 x_0}{dt^2} + (k_u + k_0) x_0 - k_0 r_1 x_1 = f(t) \\ m_1 \dfrac{d^2 x_1}{dt^2} - k_0 r_0 r_1 x_0 + k_0 r_1^2 x_1 = 0 \end{cases} \tag{4)'$$

Equation (4)' is expressed in matrix form as equation (5).

$$\begin{bmatrix} m_0 & 0 \\ 0 & m_1 \end{bmatrix} \begin{bmatrix} \ddot{x}_0 \\ \ddot{x}_1 \end{bmatrix} + \begin{bmatrix} k_u + k_0 & -k_0 r_1 \\ -k_0 r_1 & k_0 r_1^2 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \end{bmatrix} = \begin{bmatrix} f(t) \\ 0 \end{bmatrix} \tag{5}$$

We discretized equation (5) using central difference formula, then we get equation (6), where,

δ: length of time in each step [s]

j: discretized time [-]

Displacement is defined as follows:

$$x_0 = x_0^j$$

$$x_1 = x_1^j$$

$$\frac{1}{\delta^2} \begin{bmatrix} m_0 & 0 \\ 0 & m_1 \end{bmatrix} \begin{bmatrix} x_0^{j+1} - 2x_0^j + x_0^{j-1} \\ x_1^{j+1} - 2x_1^j + x_1^{j-1} \end{bmatrix} + \begin{bmatrix} k_u + k_0 & -k_0 r_1 \\ -k_0 r_1 & k_0 r_1^2 \end{bmatrix} \begin{bmatrix} x_0^j \\ x_1^j \end{bmatrix} = \begin{bmatrix} f^j \\ 0 \end{bmatrix} \tag{6}$$

Equation (6) is rewritten as recurrence form as equation (7).

$$\begin{bmatrix} x_0^{j+1} \\ x_1^{j+1} \end{bmatrix} = \begin{bmatrix} 2 - \dfrac{\delta^2 (k_u + k_0)}{m_0} & \dfrac{\delta^2 r_1 k_0}{m_0} \\ \dfrac{\delta^2 r_1 k_0}{m_1} & 2 - \dfrac{\delta^2 r_1^2 k_0}{m_1} \end{bmatrix} \begin{bmatrix} x_0^j \\ x_1^j \end{bmatrix} - \begin{bmatrix} x_0^{j+1} \\ x_1^{j+1} \end{bmatrix} + \begin{bmatrix} \dfrac{\delta^2}{m_0} f^j \\ 0 \end{bmatrix} \tag{7}$$

Assuming initial condition and defining external force term, then we can calculate displacement values step by step.

Initial conditions are assumed as follows:

$$\begin{bmatrix} x_0^{-1} \\ x_1^{-1} \end{bmatrix} = \begin{bmatrix} x_0^0 \\ x_1^0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Let us define frequency of external force as F, then external force term is expressed as $f(t) = f_A \sin(2\pi F t)$. Central difference formula of $f(t) = f_A \sin(2\pi F t)$ is expressed as equation (8). f_A stands for amplitude [N].

$$f^j = f_A \sin(2\pi F \cdot \delta \cdot j) \tag{8}$$

Substituting equation (8) by (7) then we get

$$\begin{bmatrix} x_0^{j+1} \\ x_1^{j+1} \end{bmatrix} = \begin{bmatrix} 2 - \dfrac{\delta^2 (k_u + k_0)}{m_0} & \dfrac{k_0 r_1 \delta^2}{m_0} \\ \dfrac{k_0 r_1 \delta^2}{m_1} & 2 - \dfrac{k_0 r_1^2 \delta^2}{m_1} \end{bmatrix} \begin{bmatrix} x_0^{j} \\ x_1^{j} \end{bmatrix} - \begin{bmatrix} x_0^{j-1} \\ x_1^{j-1} \end{bmatrix} + \begin{bmatrix} \dfrac{\delta^2}{m_0} f_A \sin(2\pi F \delta \cdot j) \\ 0 \end{bmatrix} \quad (10).$$

Equation (10) is recurrence formula then we get discretized displacement values in time series.

[3] Example of Simulation

Here is an example of simulation result of FE166Sigma with recommended cabinet in the manual. Specification of FE166Sigma is given in Table1.

Table1 Specification of FE166Sigma

Parameter	Value	Note
f_0	50[Hz]	
m_0	0.0069[kg]	6.9[g]
a_0	0.013273[m^2]	Effective radius of membrane = 6.5[cm]

k_uは is calculated as:

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k_u}{m_0}} \quad (11)$$

then,

$$k_u = 4\pi^2 f_0^2 m_0 \quad =681.00[\text{N/m}] \quad (12)$$

Recommended cabinet in the manual by Fostex is:

Volume: 25ℓ=0.025[m^3]

Duct dimension (H×W×L): 60mm×110mm×135mm

Effective length of the duct is estimated adding 0.7 x effective radius of cross section cross section of duct. Then effective length of the duct is estimated as 167mm.

Hence we get

a_1 = 0.00660[m^2]

m_1 = 0.001323[kg]

$$k_0 = \frac{\gamma a_0^2 P}{V} = \frac{1.4 \times 0.01327^2 \times 101{,}300}{0.025} = 998.94\,[\text{N/m}] \quad (13)$$

We let size of time step δ=0.00001[s]. This time step can clearly express 100Hz vibration. Driving force is assumed 0.1[N]. This force value will not harm membrane. Note that this condition is not identical to that output power is constant. These conditions are seen in Table2.

Table 2 Values of Parameters

Parameter	Symbol	Value	Unit
Effective membrane area	a_0	0.01327	[m^2]
Cross sectional area of duct	a_1	0.006600	[m^2]
Ratio of duct area vs. membrane area (a_1/ a_0)	r_1	0.00497362	[-]

Effective moving mass of membrane	m_0	0.006900	[kg]
Effective moving mass of air in duct	m_1	0.001323	[kg]
Spring constant of speaker unit	k_u	681.0	[N/m]
Spring constant of chamber corresponding to membrane under adiabatic condition (equithermal condition)	k_0	998.94 (713.53)	[N/m]
Amplitude of driving force	f_A	0.1	[N]
Size of discretized time	δ	0.00001	[s]

Characteristic frequency of duct is independent of speaker unit and calculated as:

$$f_D = \frac{r_1}{2\pi}\sqrt{\frac{k_0}{m_1}} \quad =68.8[\mathrm{Hz}] \tag{14}$$

Equation (13) is based on adiabatic condition. Adiabatic condition is considered right condition; however, this result is a little bit different from Nagaoka's. Calculating using formula in Nagaoka's "Newest original 20 speaker craft works" gives 60.7[Hz] as characteristic frequency. Calculating based on equithermal condition gives 58.1[Hz] as characteristic frequency. Note that I use equithermal condition for all the calculations, because it gave better results than adiabatic condition based on my actual works of MCAP-CRs.

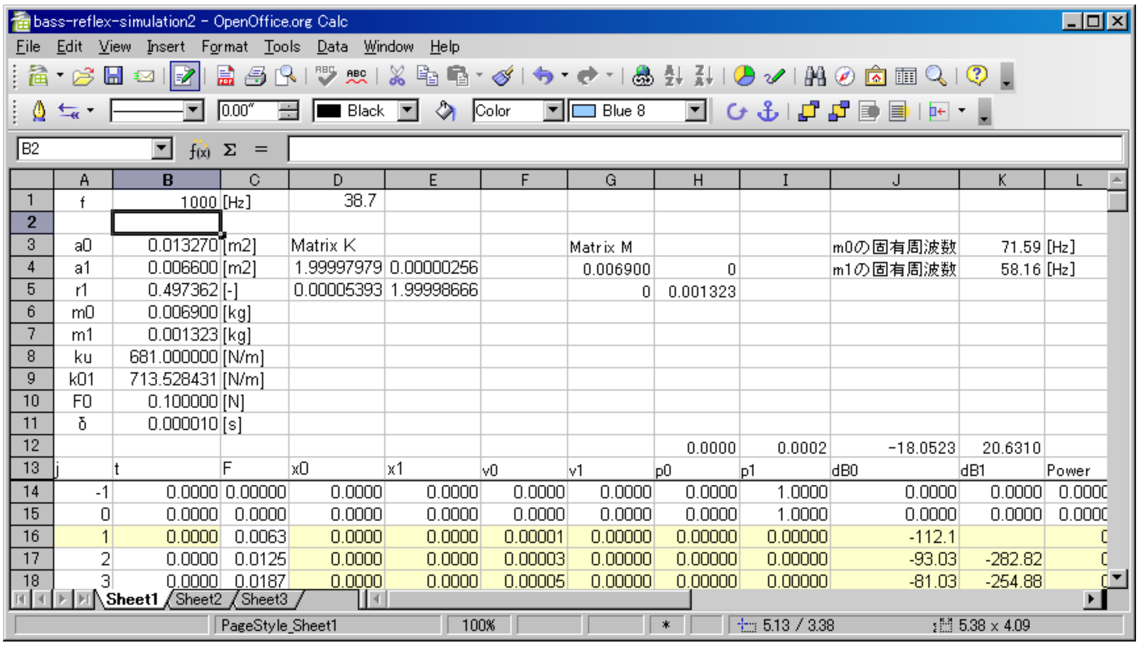
All the simulation results are based on equithermal conditions in this paper.

General spreadsheet software is enough to execute this simulation. Open Office Calc was used for this calculation.

Fig. 2 shows calculation screen of Open Office Calc in this simulation. Each parameter is defined in cells B1 - B11. D4, E4, D5 and E5 are element of matrix of right hand side of 1st term of equation (3). Cells A14 and below are discretized time step, B14 and below are time at discretized time step. C14 and below are force. D14 and below are displacement of membrane mass. E14 and below are displacement of air mass of duct.

Velocity of mass is derived from displacement and time. Sound pressure is calculated from dynamic pressure of mass, then converted to sound pressure level. H14, I14 and below are dynamic pressure [Pa] and J14, K14 and below are converted sound pressure level [dB]. Power level of driving amplifier is not known in this simulation.

D14:E15 is initial conditions. D16 and E16 are calculated using equation (7). Row 17 and below are just dragged from row 16.



	A	B	C	D	E	F	G	H	I	J	K	L
1	f	1000	[Hz]	38.7								
2												
3	a0	0.013270	[m2]	Matrix K			Matrix M			m0の固有周波数	71.59	[Hz]
4	a1	0.006600	[m2]	1.99997979	0.00000256		0.006900	0		m1の固有周波数	58.16	[Hz]
5	r1	0.497362	[-]	0.00005393	1.99998666		0	0.001323				
6	m0	0.006900	[kg]									
7	m1	0.001323	[kg]									
8	ku	681.000000	[N/m]									
9	k01	713.528431	[N/m]									
10	F0	0.100000	[N]									
11	δ	0.000010	[s]									
12								0.0000	0.0002	-18.0523	20.6310	
13	j	t	F	x0	x1	v0	v1	p0	p1	dB0	dB1	Power
14	-1	0.0000	0.00000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.000
15	0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.000
16	1	0.0000	0.0063	0.0000	0.0000	0.00001	0.00000	0.00000	0.00000	-112.1		0
17	2	0.0000	0.0125	0.0000	0.0000	0.00003	0.00000	0.00000	0.00000	-93.03	-282.82	0
18	3	0.0000	0.0187	0.0000	0.0000	0.00005	0.00000	0.00000	0.00000	-81.03	-254.88	0

Fig. 2 Calculation Window

Results of this simulation were interesting. Figs. 3A and 3B shows displacement of mass.

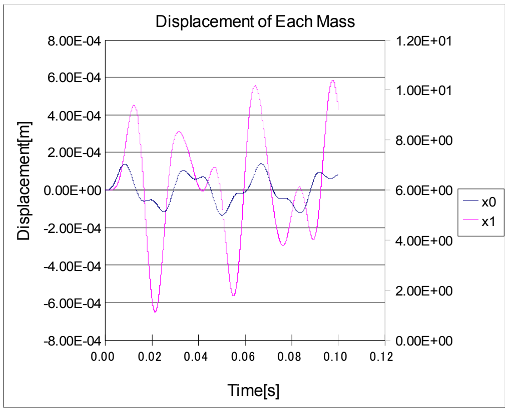


Fig. 3A Displacement of each mass in time series (58.1Hz)

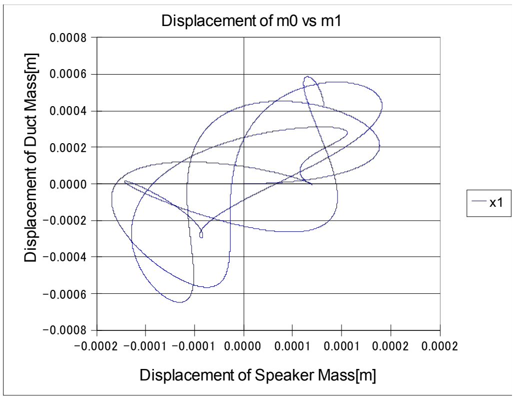


Fig. 3B Displacement of duct mass vs. membrane mass (58.1Hz)

58.1[Hz] is characteristic frequency of mass. Fig. 3A shows that at characteristic frequency, phase of membrane (blue) and phase of air mass in the duct should be reverse; however, it is not clearly shown. On the other hand, displacement of air in the duct is larger than $1/r_1$ x displacement of membrane. This means that it consists of characteristic vibration.

Fig. 3B is scatter plot of displacement of air in the duct vs. displacement of membrane. If correlation between each displacement is positive, it means each phase is similar. Fig. 3B does not clearly show if correlation is positive or negative. It is not identical to physics commonsense.

Executing same simulation at different forced vibration frequency shows bass reflex phenomenon.

Fig. 4 A/B shows result of simulation at 40Hz. It is smaller than characteristic frequency. In this case, phase of each displacement is obviously same.

Fig. 5 A/B shows result of simulation at 80Hz. It is larger than characteristic frequency. In this case, phase of each displacement is obviously reverse.
These results show function of bass reflex effects.

Fig. 6A/B shows result of simulation at 1,000Hz. It was done just to see if forced vibration at much higher frequency affect the bass reflex effects. It seems there are some characteristic frequencies, but original forced frequency (small fluctuation) is not seen in displacement of air mass in the duct. Much higher frequency than characteristic frequency is filtered by cavity resonator system.

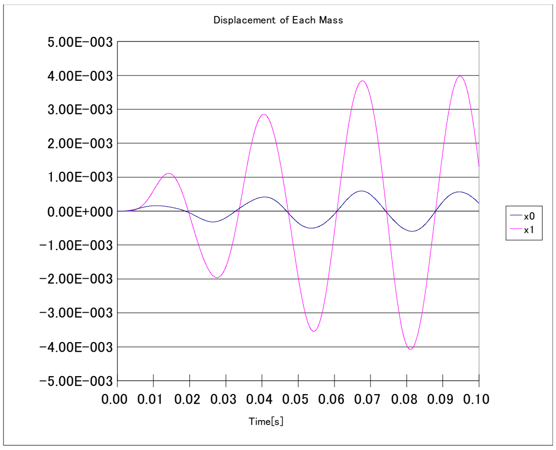


Fig. 4A (40Hz)

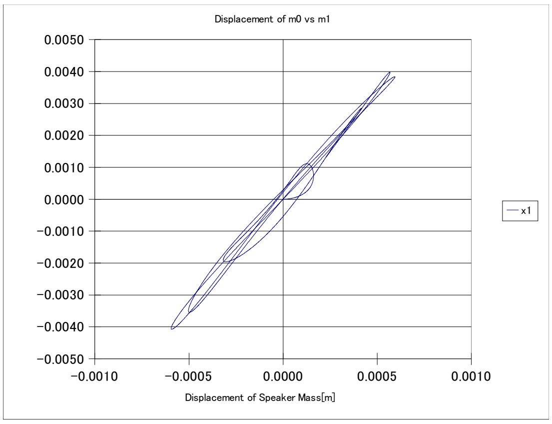


Fig. 4B (40Hz)

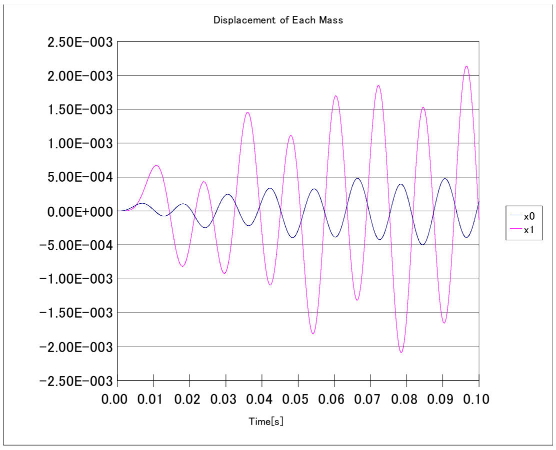


Fig. 5A (80Hz)

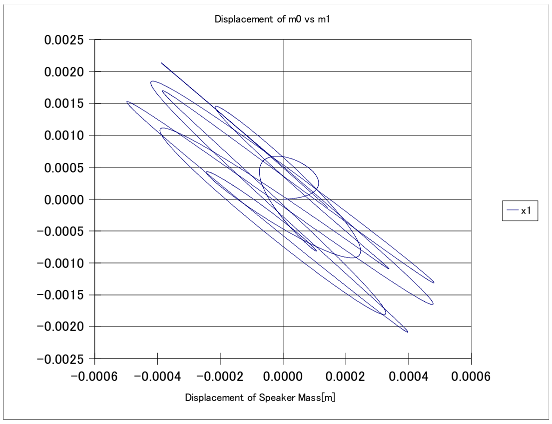


Fig. 5B (80Hz)

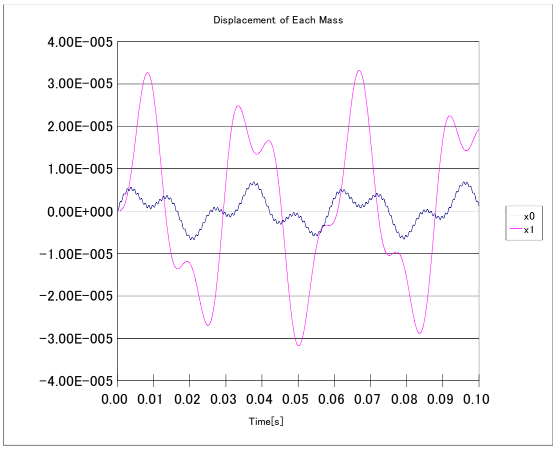


Fig. 6A (1000Hz)

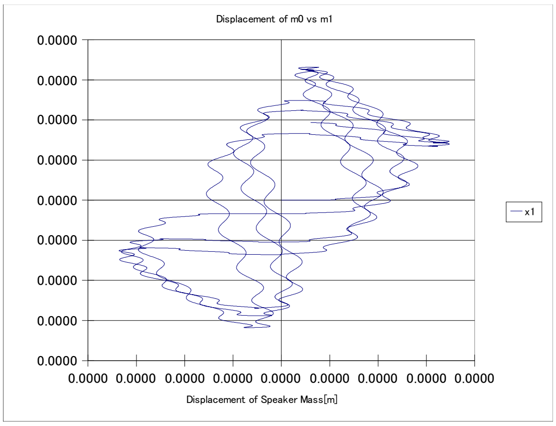


Fig. 6B (1000Hz)

Here are some more notes.

Forced vibration frequency of 40Hz results in canceling pressure wave of front radiation at this frequency. It is the reason why it is said that below characteristic frequency cannot be regenerated using bass-reflex loudspeaker system.

Forced vibration frequency of 80Hz results in that duct enforces pressure wave from front radiation. Displacement of air mass in the duct is inverse proportional to ratio of area r_1. This is bass-reflex function, but not resonator's function at characteristic frequency. When the system works as resonator, displacement of air mass is more than inverse proportional to ratio of area r_1.

When we see blue and pink lines of Fig.6A, we notice that there are 8 - 9 peaks in 0.1s. These frequencies are not related to forced frequency, so that they should be characteristic frequencies of this system. Though time lengths between next peeks are not constant, we cannot measure exact characteristic frequencies from this simulation result; however, characteristic frequencies of membrane and air mass in the duct are around 85Hz. Characteristic frequencies of this system are 71.6Hz and 58.2Hz, so that observed frequencies do not match. We need more study on this.

In order to know if there is any mistake in this simulation, difference of sound pressures between duct and membrane was calculated. Fig.7 shows level difference of sound pressures between duct and membrane vs. frequency. Fig.7 shows resonant level is higher around 50 - 60Hz. This analysis implies that this simulation is practical enough.

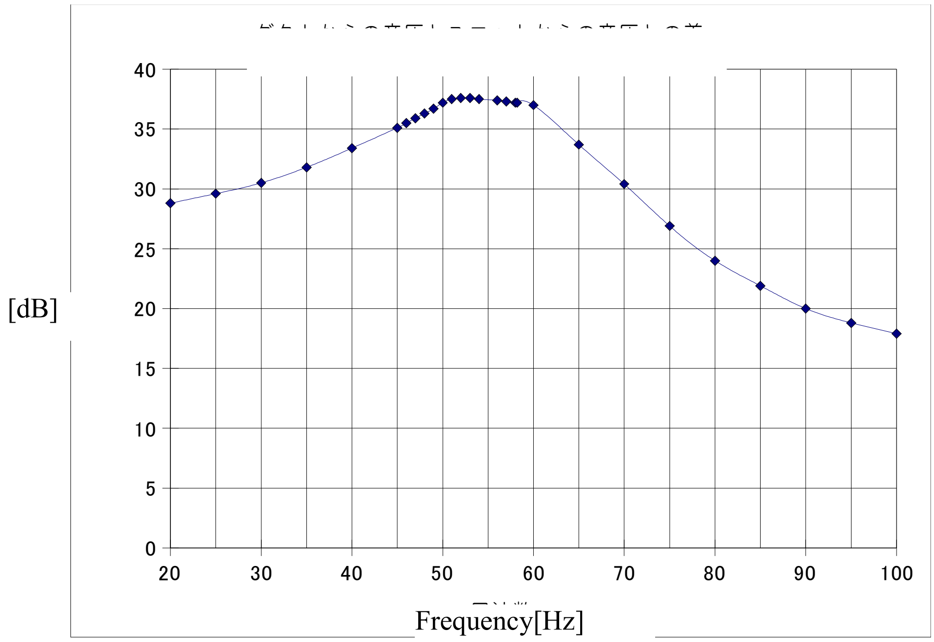


Fig. 7　Difference between sound pressures at membrane and at duct[dB]

[4] Summary

Movements of membrane and air mass in duct were analyzed solving equation of motion numerically. This method is easily extended to more multiple degree of freedom cavity resonators including MCAS, MCAP, and any AICC[1] cavity resonators.

End of Report

[1] MCAS: Multiple-Chamber Aligned in Series, MCAP: Multiple-Chamber Aligned in Parallel, AICC: Arbitrary Inter-Chamber Connection. All these names are given by Suzuki.